Direction of Arrival Estimation (DOA) with MIMO Radar with Compressive Illumination

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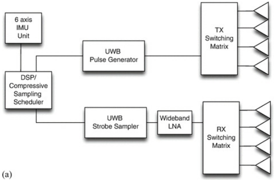
Specialists Meeting on Compressive Sensing applications for Radar/ESM and EO/IR imaging

Outline

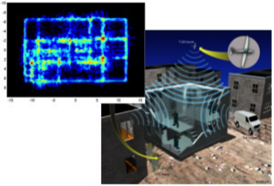
1. Wideband Multichannel Radar
   - Motivation
2. MIMO Radar
   - Problem and current approaches
   - Proposed solution
   - Problem statement
   - Simulation results
   - Hardware implementation

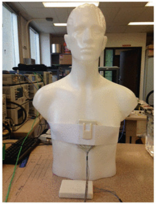
Wideband multichannel radar-Emerging applications

- Networked Sensing
- 3D/4D Imaging with Real Aperture Radar
- Massive MIMO



(a)





(b)

High Resolution radar imaging

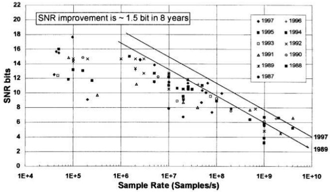
To achieve high resolution in range-angle of arrival domain, we need

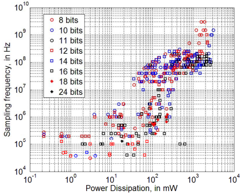
- large illumination bandwidth leads to finer range resolution $\Delta_R = \frac{c}{2B}$.
- increase in transmitter and receiver systems lead to finer angle of arrival resolution $\Delta_{cos(\theta)} = \frac{2}{N_T N_R}$

Wideband Radar Technology

Applications stretch the resolution and bandwidth capabilities of ADC technology

- COTS ADCs have limited resolution at high sampling rates
- Power consumption quadruples for additional bit of resolution [Wal99]





- Sample with available technology use signal recovery methods for high resolution
- Radar sensing is not just receive processing:
  Illumination + Receiver Filtering+Sampling
- Challenge: Design transmit waveforms and receive filtering to shift burden away from ADC

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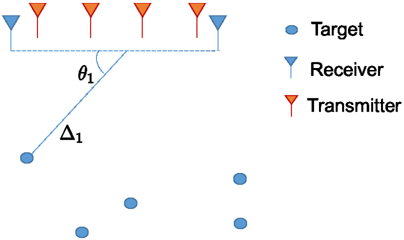
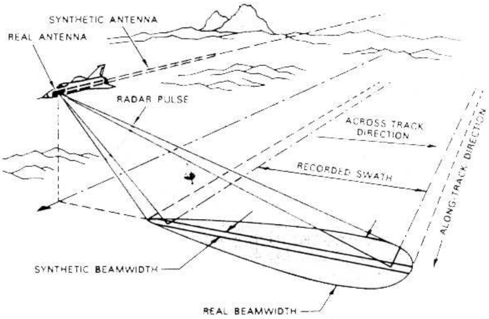
Problem

- We consider collocated multiple transmitters and receivers with a common reference.
- **Goal:** Estimate the round-trip delay Δ_i, angle of arrival θ_i, and target reflectivity x_i.
- **Approach:** Utilize compressive measurements from an incoherent domain.



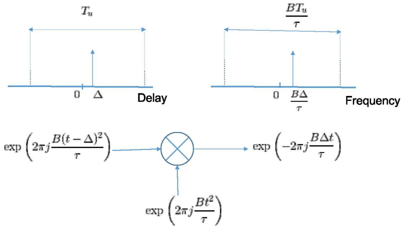
Before CS there was Stretch

- Stretch processing considers a fixed range swath.
- Uses LFM waveform on transmit and down-conversion implementing approximate match filtering.
- Converts delay estimation to tone estimation.



Stretch Processing with Chirp waveform

- Stretch processing considers a fixed range swath.
- Converts delay estimation to spectrum estimation.
- Sampling rate reduced from $B \rightarrow B\frac{T_u}{\tau}$



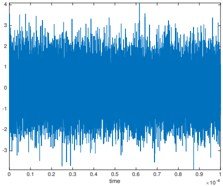
Can we design illumination schemes that further reduces sampling rate to exploit sparsity in scene?

Stochastic waveforms

Samples from a Sub-Gaussian distribution used as Tx waveforms [SNR15][1]with good theoretical guarantees [KMR14][2].

Issues

- Memory requirements
- Additional reference channel
- Power amplifier requirements due to high $PAPR = 20\log_{10}\left(\frac{|x_{max}|}{x_{rms}}\right) \approx 15dB$.



[1]M. Shastry, R. Narayanan, and M. Rangaswamy, *Sparsity-based signal processing for noise radar imaging*, IEEE Transactions on Aerospace and Electronic Systems, 2015

[2]Felix Krahmer, Shahar Mendelson, and Holger Rauhut, *Suprema of chaos processes and the restricted isometry property*, Communications on Pure and Applied Mathematics,2014.

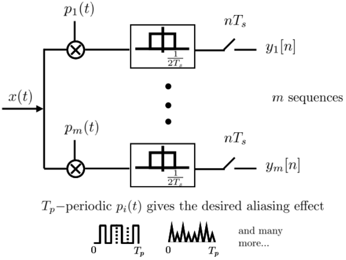
Modulated wideband converter based systems

Transmitted waveforms: Gaussian pulse[BIE14][3], FDMA and CDMA[CCEH16][4].

The periodic waveforms produce a mixed version of Fourier coefficients[GTE11][5]

Issues

- Multichannel
- Sensitivity to crystal filter response
- Parallel channels with filtering and ADC required



[4] O. Bar-Ilan and Y. C. Eldar, *Sub-nyquist radar via doppler focusing*, IEEE Transactions on Signal Processing ,2014

[5] David Cohen, Deborah Cohen, Yonina C. Eldar, and Alexander M. Haimovich, *Summer: Sub-nyquist MIMO radar*,2016

[6] K. Gedalyahu, R. Tur, and Y. C. Eldar, *Multichannel sampling of pulse streams at the rate of innovation*, IEEE Transactions on Signal Processing 2011

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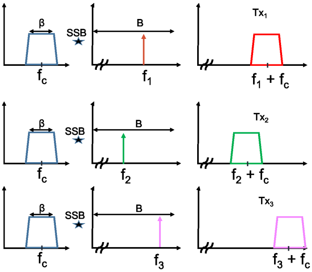
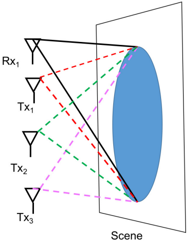
Proposed approach -Multi-frequency modulated waveform



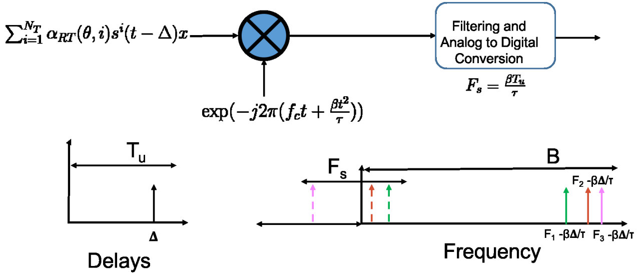


Transmitted waveform at Tx i: $s^i(t) = \exp\left(2\pi j\left(f_c t + \frac{\beta}{2\tau}t^2\right)\right) \times s_i(t)$

$$s_i(t) = \frac{1}{\sqrt{N_c N_T}} \sum_{k}^{N_c} \exp\left(2\pi j f_i(k) t\right).$$

Received Signal

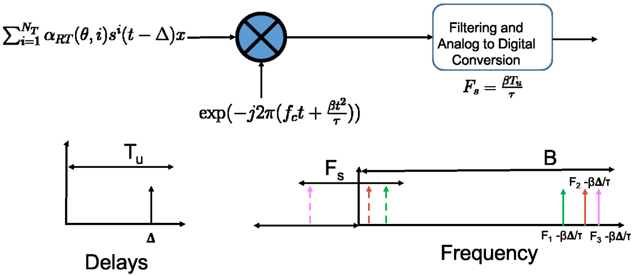
Stretch processor samples at receiver



$$y(n) = \sum_{k=1}^{N_{Targets}} \sum_{i=1}^{N_T} \sum_{c=1}^{N_c} x_k \alpha_{RT}(\theta_k, i) exp\left(2\pi j \left(f_i(c) - \frac{\beta \Delta_k}{\tau}\right) \frac{n}{F_s}\right)$$
$$\times \exp\left(2\pi j f_i(c) \Delta_k\right) + w(n)$$

Received Signal

Stretch processor samples at receiver



Stretch Processing: Range x⇒ Tone Estimation
MultiFrequency LFM: Range ⇒ Structured Line Spectrum Estimation

Discussion

Advantages

- MIMO architecture with undersampling in spatial and delay domains
- $PAPR \approx 3 + 10\log(N_c)$ dB to 1!
- Standard calibration procedures

Drawbacks

- Large analog bandwidth required for ADC
- Computational Complexity of Recovery

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Problem Definition

Given a scene with small number of targets $N_{Targets}$, we define

$$\mathbf{y} = \int\limits_{\Delta,\theta} \mathbf{\Psi}\left(\Delta,\theta\right) d\mu + \mathbf{w}.$$

$$\Psi\left(n,\Delta,\theta\right) = \sum_{i,c} \alpha_{RT}(\theta,i) exp\left(2\pi j\left(f_i(c) - \frac{\beta\Delta}{\tau}\right)\frac{n}{F_s}\right)\exp\left(2\pi j f_i(c)\Delta\right)$$

$$\mu = \sum_{k=1} x_k \delta(\Delta - \Delta_k, \theta - \theta_k).$$

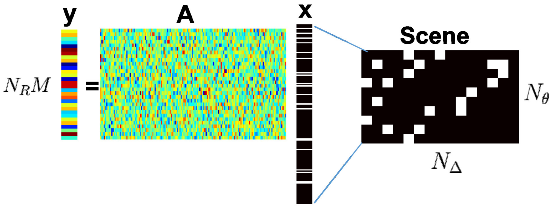
We estimate the angle of arrival θ and delay Δ by solving

$$\min_{\mu} \left\| \mathbf{y} - \int\limits_{\Delta,\theta} \mathbf{\Psi}\left(\Delta,\theta\right) d\mu \right\| \textbf{ Subject to } \left\|\mu\right\|_{TV} \le \tau, \quad (1)$$

where, $\left\|\mu\right\|_{TV} = \sum_{k} \left|x_k\right|.$

Discretization approach

- Range space is discretized with resolution $\Delta_R = \frac{c}{2B}$.
- The non-linear mapping $\cos(\theta)$ of the angle of arrival is discretized with resolution $\Delta_\theta = \frac{1}{N_T N_R}$.



$$\min_x \|\mathbf{x}\|_1 \textbf{ Subject to } \|\mathbf{Ax}-\mathbf{y}\|_2 \leq \|\mathbf{w}\|_2 .$$

Recovery Guarantees (Single TX/ RX))

Matrix Type of size $M \times N$	Mutual Coherence	Spectral Norm	Sparsity for successful recovery	Minimum signal strength	Reference
Random matrix with (NM) independent random entries	$2\sqrt{\frac{\log N}{M}}$	$\sqrt{\frac{N}{M}} + 1$	$\mathcal{O}\left(\frac{M}{\log N}\right)$	$\mathcal{O}\left(\sigma\sqrt{2\log N}\right)$	[CJ11, CP09, DS01]
Toeplitz block matrix with $(N+M)$ random entries	$\mathcal{O}\left(\sqrt{\frac{\log N}{M}}\right)$	$\mathcal{O}\left(\sqrt{\frac{N}{M}}\right)$	$\mathcal{O}\left(\frac{M}{\log N}\right)$	$\mathcal{O}\left(\sigma\sqrt{2\log N}\right)$	[Baj12]
LFM waveform modulated with $N_c \ll N$ randomly selected tones for single transmitter and receiver	$\mathcal{O}\left(\sqrt{\frac{\log N}{M}}\right)$	$\mathcal{O}\left(2\sqrt{\frac{N\log(N+M)}{M}}\right)$	$\mathcal{O}\left(\frac{M}{\log N\log(N+M)}\right)$	$\mathcal{O}\left(\sigma\sqrt{2\log N}\right)$	[SE15, SE16]

Recovery Guarantees (MIMO)

Matrix Type of size $N_R M \times N_\Delta N_\theta$	Sparsity for successful recovery	Minimum signal strength	Reference
Toeplitz block matrix with $(N_T M + N_\Delta)$ random entries	$\mathcal{O}\left(\frac{N_R M}{\log(N_\Delta N_\theta)}\right)$	$\mathcal{O}\left(\sigma\sqrt{2\log(N_\Delta N_\theta}\right)$	[Baj12][6]
LFM waveform modulated with $N_c \ll \frac{N_\Delta}{N_T}$ randomly selected tones per transmitter	$\mathcal{O}\left(\frac{N_R M}{\log^2(2N_\Delta N_\theta)}\right)$	$\mathcal{O}\left(\sigma\sqrt{2\log(N_\Delta N_\theta}\right)$	This work

[7] Waheed Bajwa, *Geometry of random toeplitz-block sensing matrices: bounds and implications for sparse signal processing*, Proc. SPIE ,2012

Continuous Domain solution

We solve the following problem

$$\min_{\mu} \left\| \mathbf{y} - \int_{\Delta,\theta} \mathbf{\Psi}\left(\Delta, \theta\right) d\mu \right\| \textbf{ Subject to } \|\mu\|_{TV} \leq \tau, \tag{2}$$

$$\text{where, } \|\mu\|_{TV} = \sum_k |x_k| .$$

We exploit the differentiability of $\mathbf{\Psi}\left(\Delta, \theta\right)$ in the parameters to refine the support [BSR17][7].

[8]Nicholas Boyd, Geoffrey Schiebinger, and Benjamin Recht, *The alternating descent conditional gradient method for sparse inverse problems*, SIAM Journal on Optimization, 2017.

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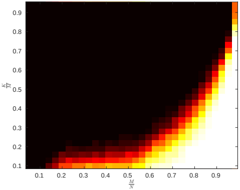
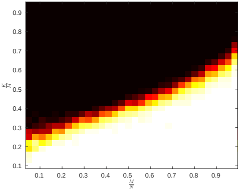
On-Grid results - Noiseless recovery

Single Tx-Rx system

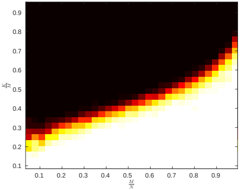
Performance criterion - $P\left(MSE < 1e-5\right)$



1 Tone



10 tones modulation

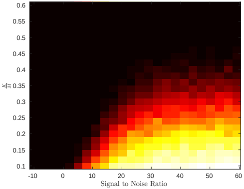


Gaussian waveform

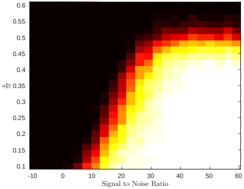
On-Grid results - Noisy support recovery

Single Tx-Rx system with $\beta/B = 0.5$

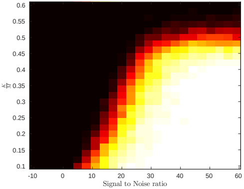
Performance criterion - $P\left(AUC > 0.99\right)$



1 Tone



10 tones modulation



Gaussian waveform

Off-grid recovery DOA- Performance metrics

We use performance metrics defined in [TBR15][8]
Given estimated model $\sum_j \hat{x}_j \Psi(\hat{\Delta}_j)$ and true model $\sum_{i=1}^{K} x_i \Psi(\Delta_i)$
$\mathcal{T} = \{\Delta_j\}$ set of true parameters
$N_{\Delta_j} = \{\Delta \in \Omega : \|\Delta - \Delta_j\| \leq 0.2c/(2B)\}$
$\mathcal{F} = \Omega \setminus \mathcal{T}$

- error due to false detections given by $m_1 = \sum_{\hat{\Delta}_i \in \mathcal{F}} |\hat{x}_i|$,
- weighted localization error $m_2 = \sum_j \sum_{i:\hat{\Delta}_i \in N_{\Delta_j}} |\hat{x}_i| \min_{\Delta \in \mathcal{T}} \left\| \hat{\Delta}_i - \Delta \right\|^2$,
- approximation error in the scattering coefficients

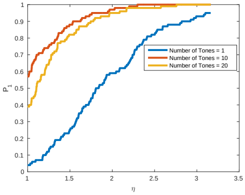
  $m_3 = \sum_{\Delta_j \in \mathcal{T}} \left| x_j - \sum_{\hat{\Delta}_l \in N_{\Delta_j}} \hat{x}_l \right|$.

[9] Gongguo Tang, B.N. Bhaskar, and B. Recht, *Near minimax line spectral estimation*, IEEE Transactions on Information Theory,2015.

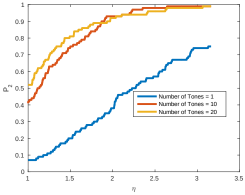
Performance comparison

Performance profile

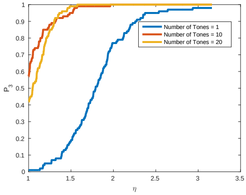
$$P_s(\eta; i) = \frac{\textbf{card}\,\{p \in \mathcal{P} : m_i(p, s) \leq \eta \min_s m_i(p, s)\}}{\textbf{card}\,\{\mathcal{P}\}}. \quad (3)$$



False detections



Localization error



Approximation error

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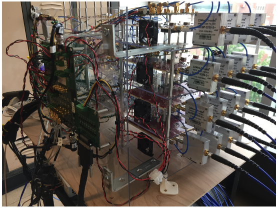
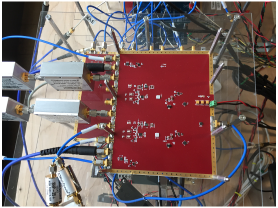
Hardware setup



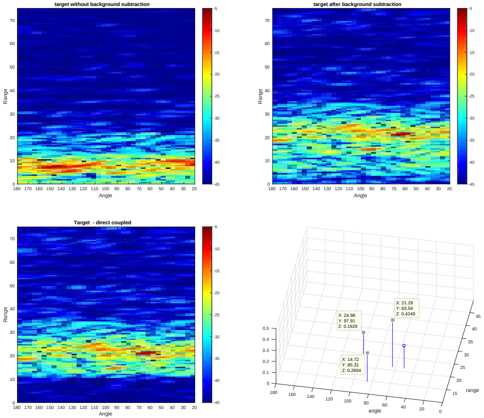


DDS Transmitter -32 **Stretch Receiver**

Hardware setup- TX, RX and antenna array

Experiment

Setup



Summary and Future Work

MIMO Radar

- Extended compressive illumination scheme to MIMO radar and spatial processing
- Established theoretical guarantees for the sampling rate requirements as a function of the sparsity of the scene.
- Simulated and Measured Data Experiments reveal accurate recovery of spars scenes.

Future Work

- Calibration for phase mismatches between channels. Key observation: Unlike classical beamformers range and spatial processing is coupled.

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Parameter choice

Parameter	Value
Bandwidth B	$500 \times 10^6 Hz$
Range Interval	$[0, 100]$m
Number of Range Bins N	334
Unambiguous time interval t_u	6.6×10^{-7} s
pulse duration τ	$6.86 \times 10^{-5} s$